DYNAMICAL SYSTEMS SYLLABUS FOR CSUREMM 2021

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Abstract: Dynamical systems are a ubiquitous element of modeling. At the center of this field is a simple fundamental question - how do local, first-principle forces, shape the overall trajectory of a system in time? In this mini-course, we will focus on ordinary differential equations (ODEs), where the state of the system is prescribed by a vector in $\mathbb{R}^d$ or $\mathbb{C}^d$. The starting point will be the following inconvenient fact: given an ODE

$$\frac{d}{dt}\underline{x}(t) = f(\underline{x}, t)\,, \qquad \underline{x}(t_0) = \underline{x}_0 \in \mathbb{C}^d\,,$$

we can rarely obtain explicit closed-form solutions. What information on the solutions can we still extract from such models?

We will briefly touch on the following topics:

- Phase space portraits and long time asymptotics
- Fixed points, stability and instability, limiting cycles
- Bifurcations
- Discrete systems and chaos
- Numerical methods and numerical study of dynamical systems

Suggested reading: My notes and tablet-whiteboard screen-shots will be distributed after the mini-course.

(1) *Strogatz, Nonlinear Dynamics and Chaos.* The most "user friendly", and best suited as an introductory text. Not much in way of proofs.

(2) *Hirsch, Smale, and Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos.* Roughly the same material, but more proofs and less emphasis on examples methods.

(3) *Coddington and Levinson, Theory of Ordinary Differential Equations.* Very clear, very good. Only partly overlaps with (2).

(4) *Arnold, Mathematical Methods of Classical Mechanics.* Very advanced, for the analysis- and geometry-oriented (probably not "get me what I need to know for the REU"-type book).

(5) *Iserles, A first course in the numerical analysis of differential equations.* Very easy introduction to numerical methods, with good proofs, but can be read without proofs.

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